Code No. 1045 / CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, February 2021

Sub: Mathematics

Paper – I : Algebra

Time: 2 Hours **Max.Marks:80**

PART – A

Answer any four questions. **(4x5=20 Marks)**

1 If G is a group and X is a G-set then prove that the action of G on X induces a homomorphism $\phi : G \to S_x$.

2 Define normal series and composition series. Give an example of each.

3 Show that any non-zero homomorphism of a field F into a ring R is one-one.

4 Prove that every Euclidean domain is a PID.

5 If R is a ring with unity then prove that an R-module M is cyclic if and only if M $\simeq \frac{R}{I}$ for some left ideal I of R.

6 Show that $x^3 - 5x + 10$ is irreducible order Q.

7 If the multiplicative group F^* of non-zero elements of a field F is cyclic then prove that F is finite.

8 Prove that the Galois group of $x^4 + 1 \in Q[x]$ is the Klein four group.

PART – B

Answer any four questions. **(4x15=60 Marks)**

9 If G is a finite group of order p^n where p is a prime and n > 0 then prove the following:
(i) G has a nontrivial center Z.
(ii) $Z \cap N$ is nontrivial for any nontrivial normal subgroup N of G.

10 State and prove second and third Sylow theorems.

11 If R is a non-zero ring with unity and I is an ideal in R such that $I \neq R$ then prove that there exists a maximal ideal M of R such that $I \subseteq M$.

12 Prove that every PID is a UFD but a UFD need not be a PID.

13 If R is a ring with unity and M is an R-module then prove that the following are equivalent.
(i) M is simple
(ii) M $\neq$ (0) and M is generated by any $x \in M$ where $x \neq 0$.
(iii) M $\simeq \frac{R}{I}$ where I is a maximal left ideal of R.

14 If E is an algebraic extension of a field F and $\sigma : F \to L$ is an embedding of F into an algebraically closed field L then prove that σ can be extended to an embedding η: E→ L.

15 State and prove the fundamental theorem of algebra.

16 State E be a finite extension of F and $f : G \to E^*$ where $E^* = E - \{0\}$ has the property that $f(\sigma\eta) = \sigma(f(\eta))f(\sigma)$ for all $\sigma, \eta \in G$. Then prove that there exists $\alpha \in E^*$ such that $f(\sigma) = \sigma(\alpha^{-1})\alpha$ for all $\sigma \in G$.

17 State and prove Burnside theorem.

18 In a commutative ring R prove that an ideal P is prime if and only if $ab \in P$, $a, b \in R$ implies $a \in P$ or $b \in P$.

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Code No. 1063 / CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, February 2021

Sub: Statistics

Paper – I: Mathematical Analysis & Linear Algebra

Time: 2 Hours **Max.Marks:80**

PART – A

Answer any four questions. **(4x5=20 Marks)**

1 Define function of bounded variation.

2 Show that the set of points of discontinuity of a monotonic function f(x) defined on [a,b] is atmost countable.

3 Explain differentiability at a point.

4 Evaluate $f(x,y) = \int_o^1 \int_o^1 \frac{x-y}{x+y} dx\ dy$.

5 For any matrix A, show that $(A'A)^{+} = A^{+}(A')^{+}$.

6 Define Moore-Penrose inverse.

7 Show that the matrices A, $P^{-1}AP$ have the same characteristic roots when P is a non-singular matrix.

8 Define algebraic and geometric multiplicity of characteristic roots.

PART – B

Answer any four questions. **(4x15=60 Marks)**

9 Let α be a function of bounded variation on [a,b] and assume that $f \in R(a)$. on [a,b]. Then show that $f \in R(a)$ on every subinterval [c,d] of [a,b].

10 If $f \in R(a)$, then show that $\alpha \in R(\alpha)$ on [a,b] and

$$\int_a^b f(x)d\alpha(x) + \int_a^b \alpha(x)df(x) = f(b)\alpha(b) - f(a)\alpha(a)$$

11 State and prove mean value theorem for two variable functions.

12 State and prove Tailors theorem for two variables.

13 Define orthogonal and unitary matrix. Show that column vectors / row vectors of a unitary matrix are normal and orthogonal in pairs.

14 Describe Gram-Schmidt orthogonalization process.

15 Derive spectral decomposition of a real symmetric matrix.

16 Define quadratic forms. State and prove properties of congruent matrices and congruent quadratic forms.

17 Define rank, index and signature of a quadratic forms. Explain classification of quadratic forms.

18 State and prove:

a) Cauchy-Schwartz inequality and

b) Hadamard's inequality.

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Code No. 1048 / CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, February 2021

Sub : Mathematics

Paper – IV: Elementary Number Theory

Time: 2 Hours **Max.Marks:80**

PART – A

Answer any four questions. **(4x5=20 Marks)**

1 Show that there are infinitely many primes.

2 Prove that $\phi^{-1}(n) = \sum_{d/n} d\,\mu(d)$.

3 If $c > 0$ then prove that $a \equiv b$ (mod m) if and only if $ac \equiv bc$ (mod mc).

4 If P is an odd prime then prove that $1^{P-1} + 2^{P-1} + \dots + (P-1)^{P-1} \equiv (-1) \pmod{P}$.

5 Find the quadratic residues modulo 17.

6 Prove that Legendre symbol is completely multiplicative.

7 Prove that $\left(\frac{a^2 n}{P}\right) = \left(\frac{n}{P}\right)$ whenever (a, p) = 1 and P is an odd integer.

8 Evaluate $\left(\frac{219}{383}\right)$.

PART – B

Answer any four questions. **(4x15=60 Marks)**

9 (i) Prove that $d(n)$ is odd if and only if n is a square.

(ii) Prove that $\prod_{t/n} t = n^{\frac{d(n)}{2}}$.

10 State and prove generalized Mobius inversion formula.

11 (i) For $n \geq 1$, show that $\sum_{d/n} \phi(d) = n$.

(ii) For $n \geq 1$, show that $\phi(n) = \sum_{d/n} \mu(d)\frac{n}{d}$.

12 If both g and $f*g$ are multiplicative then prove that f is multiplicative.

13 State and prove Lagrange's theorem for polynomial congruences.

14 Prove that the quadratic congruence $x^2+1 \equiv 0$ (mod P) where P is an odd prime has a solution if and only if P$\equiv$1 (mod 4).

15 State and prove Gauss Lemma.

16 Let P be an odd prime and d > 0 be such that d divides (P-1). Then prove that in every reduced residue system modulo P there are exactly $\varphi(d)$ numbers "a" such that $\exp_P(a) = d$.
Also prove that there are exactly $\phi(P-1)$ primitive roots modulo P.

17 State and prove Euler's recursion formula for p(n).

18 For complex z and x with |x| < 1 and z≠0, prove that

$$\prod_{n=1}^{\infty}\left(1-x^{2n}\right)\left(1+x^{2n-1}z^{2}\right)\left(1+x^{2n-1}z^{-2}\right)=\sum_{m=-\infty}^{\infty}x^{m^2}z^{2m}.$$

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Code No. 1054 / CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, February 2021

Sub: Mathematics

Paper – IV : Complex Analysis

Time: 2 Hours **Max.Marks:80**

PART – A

Answer any four questions. **(4x5=20 Marks)**

1 Define analytic function. Verify Cauchy-Reimann equations for the functions of z^2 and z^3.

2 Prove that a linear transformation carries circles into circles.

3 Compute $\int_{|z|=2} \frac{dz}{z^2+1}$.

4 Prove that z = 0 is not a removable singular point of $f(z) = \frac{\sin z}{z^2}$.

5 If $f(z) = \frac{(3z+1)^4}{(z-1)^2(z-3)^4}$, then compute the value of $\int_{|z|=4} \frac{f'(z)}{f(z)} dz$.

6 Evaluate the poles and residue at those poles of $f(z) = \frac{z^2}{(z-1)^2(z-2)}$.

7 Show that $\prod_{n=2}^{\infty} \left(1 - \frac{1}{n^2}\right) = \frac{1}{2}$.

8 Show that $\Gamma\left(\frac{1}{6}\right) = 2^{-1/2}\left(\frac{3}{\pi}\right)^{1/2}\left(\frac{\pi}{3}\right)^2$.

PART – B

Answer any four questions. **(4x15=60 Marks)**

9 State and prove the sufficient condition for analytic function.

10 Prove that the cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or on a straight line.

11 State and prove Cauchy's integral formula.

12 Evaluate $\int_{|z|=e} \frac{|dz|}{|z-a|^2}$.

13 State and prove Cauchy's residue theorem.

14 State and prove the mean value property for harmonic functions.

15 State and prove Weierstrass theorem.

16 State and prove Legendre's duplication formula.

17 Compute $\int_0^{2\pi} \frac{d\theta}{3+2\cos\theta}$ using residues.

18 State and prove Jensen's formula.

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Code No. 1066 / CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, February 2021

Sub : Statistics

Paper – IV : Sampling Theory & Theory of Estimation

Time: 2 Hours **Max.Marks:80**

PART – A

Answer any four questions. **(4x5=20 Marks)**

1 Explain the method of drawing a random sample using systematic sampling procedure.

2 What are sampling and non-sampling errors?

3 Explain the cumulative total method of drawing PPS sample with replacement.

4 Compare PPSWOR with SRSWOR.

5 If $x_1, x_2 \ldots x_n$ is a random sample from a $N(\mu, 1)$ population then show that $t = \frac{1}{n}\sum_{i=1}^{n} x_i^2$ is an unbiased estimator of $\mu^2 + 1$.

6 Explain the concepts of:
i) LMUVE and
ii) UMVUE
with suitable examples.

7 Let X ~ N $(0, \theta)$, show that T(x) = x^2 is complete.

8 Let $x_1, x_2 \ldots x_n$ be a random sample of size n drawn from exponential population with density $f(x, \theta) = \theta e^{-\theta x}$, $x \geq 0,\ \theta > 0$ then find the MLE of θ.

PART – B

Answer any four questions. **(4x15=60 Marks)**

9 If the population consists of a linear trend, then show that $V(\overline{Y}_{St}) \leq V(\overline{Y}_{Sys}) \leq V(\overline{Y}_n)_{SRS}$.

10 Prove that in SRSWOR, sample mean is an unbiased estimator of population mean and also obtain the variance of sample mean.

11 Derive Horwitz-Thompson estimator for the population total and find its variance. Also find Yates and Grundy variance estimator.

12 Derive the variance of Regression estimator of population mean in SRS with
i) Pre assigned value of regression coefficient and
ii) Estimated value of regression coefficient.

13 State and prove Cramer-Rao inequality and explain its role in the theory of estimation.

14 Define unbiased estimator. If $x_1, x_2, \ldots, x_n$ ~ N(u, σ^2) then prove that sample mean is an unbiased estimator of μ. Also obtain the unbiased estimator for the population variance.

15 Explain the method of moments for estimating parameters of normal distribution. Comment on the efficiency of these estimators with respect to maximum likelihood estimators.

16 Define:
i) Interval estimation and
ii) Confidence level.
Explain the Pivot method of obtaining a confidence interval. Derive the confidence interval for the parameter μ when the sample is drawn from N(μ, σ^2).

17 In cluster sampling, find an unbiased estimator for the population mean and derive its variance. Also find the relative efficiency of this estimator over SRSWOR.

18 Define complete sufficient statistic and explain its importance in estimation with suitable examples. State and prove Lehman – Scheffe theorem.

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Code No. 1046 / CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, February 2021

Sub: Mathematics

Paper – II: Real Analysis

Time: 2 Hours **Max.Marks:80**

PART – A

Answer any four questions. **(4x5=20 Marks)**

1 Define a countable set. Prove that the set Z of all integers is a countable set.

2 Prove that every compact subset of a metric space is closed.

3 Define Cauchy's product of two series. Give an example to show that Cauchy's product of two convergent series need not be convergent.

4 Suppose f is a continuous function defined on a compact metric space X into a metric space Y. Prove that f(X) is a compact subset of Y.

5 If P^* is a refinement of P with usual notations, prove that $L(P, f, \alpha) \leq L(P^*, f, \alpha)$.

6 If $f \in R(\alpha)$ on $[a, b]$ and c is a constant, prove that $cf \in R(\alpha)$ on $[a, b]$ and $\int_a^b (cf)\, d\alpha = c\int_a^b f\, d\alpha$.

7 State and prove Cauchy's criteria for uniform convergence of a sequence of functions.

8 Suppose X is a finite dimensional vector space and A is a linear operator on X. Prove that A is one-one if and only if range of A=X.

PART – B

Answer any four questions. **(4x15=60 Marks)**

9 i) Suppose $\{K_\alpha\}$ is a collection of compact subsets of a metric space such that intersection of any finite sub collection is non-empty. Prove that $\bigcap_\alpha K_\alpha$ is non-empty.

ii) Prove that every k-cell is compact in R^k.

10 Prove that a subset E of R is connected if and only if E is an interval.

11 State and prove Riemann's rearrangement theorem.

12 Suppose (X, d) and (Y, ρ) are metric spaces and f: X→ Y is a mapping. Prove that f is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

13 Prove that $f \in R(\alpha)$ on $[a, b]$ if and only if given any ϵ>0 there exists a partition P of $[a, b]$ such that $U(P, f, \alpha) - L(P, f, \alpha) < \in$.

14 Suppose γ is a curve on [a, b] such that γ' is continuous on [a, b]. Prove that γ is rectifiable and $\wedge(\gamma) = \int_a^b |\gamma'(t)|\, dt$.

15 Suppose $f_n \to f$ uniformly on a set E in a metric space X. Let x be a limit point of E. Suppose $\lim_{t \to x} f_n(t) = A_n$ for every n . Then prove that the sequence $\{A_n\}$ converges. Also prove that $\lim_{n \to \infty} A_n = \lim_{t \to x} f(t)$.

16 State and prove contraction principle.

17 Prove that Cantor's set is (i) compact and (ii) perfect.

18 Prove that on a non-compact bounded subset of R there exists a continuous function which is not uniformly continuous.

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Code No. 1064 / CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, February 2021

Sub: Statistics

Paper – II: Probability Theory

Time: 2 Hours **Max.Marks:80**

PART – A

Answer any four questions. **(4x5=20 Marks)**

1 Define De-Morgan's rules for compliments.
2 State Axiomatic definition of probability.
3 State uniqueness theorem of characteristic function with example.
4 Define convergence in probability and convergence in law.
5 State Kolomogorou's strong law of large numbers for i.i.d random variables and discuss its applications.
6 Define SLLNs, WLLNs and CLT.
7 Define Markov chain, Time homogenous Markov chain and one-step transition probability matrix.
8 Define recurrent and transient states.

PART – B

Answer any four questions. **(4x15=60 Marks)**

9 i) Prove that the distribution function of a random variable x is non-decreasing, continuous on the right with $F_x(-\infty) = 0$ and $F_x(+\infty) = 1$. Conversely, every function F with the above properties is the distribution function of a random variable on some probability space.

ii) Let the three dimensional vector $\underset{\sim}{X}$ = (X_1 X_2 X_3) has p.d.f.

$$f_x(x_1\ x_2\ x_3) = \begin{cases} 6x_1\ x_2\ x_3 & if\quad 0 \le x_1 \le 1,\ 0 \le x_2 \le 1,\ 0 \le x_3 \le 2 \\ 0 & \text{Otherwise} \end{cases}$$

Find the marginal p.d.t. of x_1, x_2, x_3 and (x_1, x_3)/

10 i) State and prove Minikowski inequality.
ii) Define expectation and show that, if x and y are one-dimensional random variables then E(x$\pm$y) = E(x) $\pm$E(y).

11 i) Define characteristic function. Show that the characteristic function of a normal distributive in $e^{itu} - \frac{t^2\sigma^2}{2}$.

ii) Show that for a characteristic function $|\varphi(t)| \le \varphi(0) = F(+\infty) - F(-\infty)$ and $\varphi(t) = \overline{\varphi}(t),$ where $\overline{\varphi}(t)$ is complex conjugate of $\varphi(t)$.

12 i) State and prove Borel strong law of large numbers.
ii) State and prove Khintchines weak law of large numbers.

13 State and prove Liapunov forms of central limit theorem.

14 State Lindeberg – Feller form of central limit theorem and discuss its applications.

15 i) Discuss classification of Stochastic process with examples.
ii) State and prove Chapman – Kolmogon equation.

16 $\{x_n, n \geq 0\}$ be a M.C. defined on the state space (0, 1, 2) with initial distribution $p\{x_o = i\} = 1/3$, i = 0, 1, 2 and with t.p.m.

$$p = \begin{matrix} 0 \\ 1 \\ 2 \end{matrix} \begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{pmatrix}$$

Find:

i) $p\{x_2 = 2, x_1 = 1, x_o = 2\}$

ii) $p(x_2 = 1 / x_o = 2)$

iii) $p(x_3 = 2 / x_o = 1)$.

17 i) Show that if state j is persistent or recurrent then $\sum_{n=0}^{\infty} p_{jj}^{(n)} = \infty$ or $< \infty$.

ii) Show that if state k is either transient or null persistent then, for every j $p_{jk}^{(n)} \to 0$ as $n \to \infty$ and state k is a periodic, persistent non-null then $p_{jk}^{(n)} \to \frac{F_{jk}}{\mu_{k\mu}}$ as $n \to \infty$.

18 Show that if state j is persistent non-null then

i) $p_{jj}^{(nt)} \to \frac{t}{\mu_{jj}}$ as $n \to \infty$ and when state j is periodic with period t.

ii) $p_{jj}^{(n)} \to \frac{1}{\mu_{jj}}$ when state j is a periodic..

iii) $p_{jj}^{(n)} \to 0$ when state j is persistent null.

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Code No. 1047 / CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, February 2021

Sub : Mathematics

Paper – III : Topology & Functional Analysis

Time: 2 Hours **Max.Marks:80**

PART – A

Answer any four questions. **(4x5=20 Marks)**

1 Let T_1 and T_2 be two topologies on a non-empty set X, then prove that $T_1 \cap T_2$ is also a topology on X.

2 Show that continuous image of compact space is compact.

3 Show that compact subspace of Hausdorff space is closed.

4 Show that components of a totally disconnected space are its points.

5 If X is a vector space define

i) Algebraic dual space X^*

ii) Second algebraic dual space X^{**}

iii) Canonical mapping of X into X^{**}.

6 Show that dual space X' of a normed space X is a Banach space.

7 Let H be a Hilbert space and U: H $\rightarrow$ H and V: H $\rightarrow$ H be unitary. Then prove that

i) U is isometric

ii) $||U|| = 1$

iii) UV is unitary.

8 Let H_1 and H_2 be Hilbert spaces and S: $H_1 \rightarrow H_2$ and T: $H_1 \rightarrow H_2$ be bounded linear operators. Then prove that:

(i) $(T^*)^* = T$

(ii) $\|T^*T\| = \|TT^*\| = \|T\|^2$

(iii) $(ST)^* = T^*S^*$

PART – B

Answer any four questions. **(4x15=60 Marks)**

9 a) Show that a metric space is sequentially compact if and only if it has the Bolzano-Weierstrass property.

b) Show that every compact metric space has the Bolzano-Weierstrass property.

10 a) State and prove Tychonoff' theorem.

b) Show that every closed and bounded subspace of $\mathbb{R}^n$ is compact.

11 Let X be a normal space and let A and B be disjoint closed subspaces of X. If $[a, b]$ is any closed interval on the real line then prove that there exists a continuous real function f defined on X, all of whose values lie in $[a, b]$ such that $f(A) = a$ and $f(B) = b$.

12 a) Let X be a topological space. If $\{A_i\}$ is a non—empty class of connected subspaces of X such that $\bigcap_i A_i \neq \phi$, then prove that $A = \bigcup_i A_i$ is also a connected space of X.

b) Let X be a compact T_2-space. Then prove that X is totally disconnected if and only if it has an open base whose sets are also closed.

13 State and prove Baire's category theorem.

14 State and prove uniform boundedness theorem.

15 Let H_1 and H_2 be Hilbert spaces and $h: H_1 \times H_2 \rightarrow K$ be a bounded sesquilinear form. Then prove that h has a representation $h(x, y) = \langle Sx, y \rangle$ where $S: H_1 \rightarrow H_2$ is a bounded linear operator. Also prove that S is uniquely determined by h and has norm $\| S \| = \| h \|$.

16 Let $T: H_1 \rightarrow H_2$ be bounded linear operator where H_1 and H_2 are Hilbert spaces. Then prove that T^*, the Hilbert adjoint operator of T is unique and is bounded linear operator with $\|T^*\| = \|T\|$.

17 Let X be an inner product space and $M \neq \phi$, a convex subset which is complete (in the metric induced by the inner product). Then prove that for every given $x \in X$ there exists a unique $y \in M$ such that $\delta = \inf_{\bar{y} \in M} \|x - \bar{y}\| = \|x - y\|$.

18 (a) If Y is a closed subspace of a Hilbert space H then prove that $Y = Y^{\perp\perp}$.
(b) For any subset $M \neq \phi$ of a Hilbert space H, prove that span of M is dense in H if and only if $M^{\perp} = \{0\}$.

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Code No. 1049 / CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, February 2021

Sub : Mathematics

Paper – III : Mathematical Methods

Time: 2 Hours **Max.Marks:80**

PART – A

Answer any four questions. **(4x5=20 Marks)**

1 If n is positive integer, show that $J_{-n}(x) = (-1)^n J_n(x)$.

2 Find the solution of $y'' - y = x$ using power series method.

3 Find the fundamental matrix for the system $x' = Ax$, where A = $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$.

4 Define Wronskian of n-functions $\phi_1, \phi_2, \ldots \phi_n$. Show that the functions $x_1(t) = \sin t$, $x_2(t) = \cos t$ are linearly independent on $-\infty < t < \infty$.

5 Define Green's function.

6 Solve the IVP $x' = 3t^2x$, $x(0) = 1$ using successive approximations method.

7 Solve: $(D^2 + 2DD' + D'^2)z = e^{3x-4y}$.

8 Solve: $p^2+q^2 = x+y$.

PART – B

Answer any four questions. **(4x15=60 Marks)**

9 State and prove orthogonality property of Legendre polynomials.

10 Show that $e^{2xt-t^2} = \sum_{n=0}^{\infty} \frac{H_n(x)\, t^n}{n!}$.

11 Let $b_1, b_2, \ldots, b_n$ be defined and continuous on an interval I. Then show that any set of n solutions $\phi_1, \phi_2, \ldots \phi_n$ of $L(x) = x^{(n)} + b_1(t)x^{(n-1)} + \ldots + b_n(t)\, x = 0$ on I are linearly independent on I if and only if $w(t) = w(\phi_1, \phi_2, \ldots \phi_n)(t) \neq 0$ for all t in I.

12 Let $b_1, b_2, \ldots, b_n$ be continuous on an interval I. Let $\phi_1, \phi_2, \ldots \phi_n$ be a basis for the solutions of $L(x) = x^{(n)} + b_1(t)\, x^{(n-1)} + \ldots + b_n(t)x = 0$ *M*. Then show that a particular solution ψ_p (t) of that equation $L(x) = x^{(n)} + b_1(t)x^{(n-1)} + \ldots + b_n(t)x = h(t)$ is given by

$$\psi_p(t) = \sum_{k=1}^{n} \varphi_k(t) \int_{t_0}^{t} \frac{w_2(s)h(s)}{w(s)}\, ds.$$

13 State and prove contraction principle.

14 Let f be continuous function defined on the rectangle $R: |t - t_0| \leq a, |x - x_0| \leq b, (a, b > 0)$. Then show that a function ϕ is solution of the IVP $x' = f(t,x), x(t_0) = x_0$ on an interval I containing the point t_0 if and only if it is solution of the integral equation $x(t) = x_0 + \int_{t_0}^{t} f(s, x(s))ds$.

15 Find the complete integral of $z^2(p^2z^2+q^2)=1$ also find the singular integral if exists.

16 Solve (i) $(x^2-yz)\,p+(y^2-zx)q = z^2 - xy$ and (ii) $p^2+q^2=1$.

17 Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \cos mx \cos ny$.

18 Solve r + s – 6t = y cosx.

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Code No. 1065 / CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, February 2021

Sub: Statistics

Paper – III: Distribution Theory & Multivariate Analysis

Time: 2 Hours **Max.Marks:80**

PART – A

Answer any four questions. **(4x5=20 Marks)**

1 Derive moment generating function of two parameter gamma distribution.
2 Explain Weibul distribution ad find its mean.
3 Define compound binomial distribution and derive it's mean.
4 Define truncated Poisson and normal distributions. Give an illustration in each case.
5 Define Wishart distribution and establish its additive property.
6 Establish the relationship between multiple and partial correlation coefficients and explain their significance.
7 Define the first two principle components and show that the sum of the variances of all principal components is equal to the sum of the variances of all original variables.
8 Briefly explain the single linkage method.

PART – B

Answer any four questions. **(4x15=60 Marks)**

9 i) Derive moment generating function of normal distribution and hence find the mean and variance.

ii) If $X \sim N(\mu, \sigma)$, then show that $Y = \frac{(X - \mu)^2}{2\lambda\sigma^2}$ is gamma $(\lambda, 1/2)$.

10 i) Derive the characteristic function of Beta distribution of second kind and hence or otherwise find its mean and variance.

ii) Define Cauchy distribution. If X follows Cauchy distribution, then find the p.d.f. for x^2 and identify its distribution.

11 If $\overline{X}$ and S^2 are the sample mean and sample variance based on a random sample from a normal population, then derive their sampling distributions and show that they are independent.

12 i) X, Y are independent uniform variables over (0,1). Find the p.d.f of 1) U = XY and 2) V = X/Y.

ii) If X,Y are iid exponential random variables with parameter θ, then find the distribution of V=X / (X+Y).

13 Show that the sample mean vector and dispersion matrix of the of the multi variate normal population are independently distributed and derive their sampling distributions.

14 Define multivariate normal distribution. Prove that the conditional distribution obtained from the multivariate normal distribution is also multivariate normal.

15 Derive linear discriminant function and hence describe the classification between two multivariate populations.

16 Write in detail about multi-dimensional scaling.

17 Obtain the ML estimators of the parameters of a multivariate normal distribution.

18 i) Define k-parameter exponential family distribution and express the distributions normal and beta 2^{nd} kind in the form of exponential family if exists.

ii) Define t and F distributions and state their properties and give their applications.

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