PROF. G. RAM REDDY CENTRE FOR DISTANCE EDUCATION

DEPARTMENT OF MATHEMATICS

OSMANIA UNIVERSITY

M.Sc. Mathematics Syllabus

Semester – I & II

(Choice Based Credit System)
(w.e.f. the academic year 2023-2024)

M. Sc. Mathematics

M 101 **Semester-I**

Paper-I: Abstract Algebra

Unit- I
Automorphisms - Conjugacy and G - sets - Normal series - Solvable groups - Nilpotent groups.
(Page No. 104 to 128)

Unit- II
Structure theorems of groups: Direct products - Finitely generated abelian groups - Invariants of a finite abelian group - Sylow theorems - Groups of orders p^2, pq.
(Page No. 138 to 155)

Unit- III
Ideals and homomorphisms - Sum and direct sum of ideals, Maximal and Prime ideals - Nilpotent and nil ideals - Zorn's lemma.
(Page No. 179 to 211).

Unit- IV
Unique factorization domains - Principal ideal domains - Euclidean domains - Polynomial rings over UFD - Rings of Fractions.
(Page No. 212 to 228)

Text Book:

- **Basic Abstract Algebra** by P.B. Bhattacharya, S.K. Jain and S.R. Nagpaul. Second Edition

References:

1. **Topics in Algebra** by I.N. Herstein.
2. **Elements of Modern Algebra** by Gibert and Gilbert.
3. **Abstract Algebra** by Jeffrey Bergen.
4. **Basic Abstract Algebra** by Robert B Ash.

M. Sc. Mathematics

M 102 **Semester-I**

Paper - II: Mathematical Analysis

Unit- I
Metric spaces - Compact sets - Perfect sets - Connected sets.
(Page No. 30-46)

Unit- II
Limits of functions - Continuous functions - Continuity and compactness, Continuity and connectedness - Discontinuities - Monotonic functions, Differentiation.
(Page No. 83-102)

Unit- III
Riemann - Steiltjes integral - Definition and Existence of the Integral - Properties of the integral – Integration and differentiation, Integration of vector valued functions - Rectifiable curves.
(Page No. 120-133 & 135-142)

Unit- IV
Sequences and Series of Functions: Uniform convergence - Uniform convergence and continuity - Uniform convergence and integration - Uniform convergence and differentiation – The Stone-Weierstrass theorem.
(Page No. 143-154, 159-161, 165-171 & 220-222)

Text Book:

- **Principles of Mathematical Analysis** (3rd Edition) By Walter Rudin, *McGraw-Hill International Edition.*

References:

1. **The Real Numbers** by John Stillwel.
2. **Real Analysis** by Barry Simon.
3. **Mathematical Analysis** Vol - I by D J H Garling.
4. **Measure and Integral** by Richard L.Wheeden and Antoni Zygmund.

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M 103 **Semester-I**

Paper - III: Ordinary Differential Equations

Unit- I

Existence and Uniqueness of Solutions: Preliminaries – Successive approximations – Picard's theorem – Some examples – Continuation and dependence on initial conditions – Existence of solutions in the large – Existence and uniqueness of solutions of systems.

Unit- II

Linear Differential Equations of Higher Order: Introduction – Higher order linear differential equations – A Mathematical model – Linear dependence and Wronskian – Homogeneous linear equations with constant coefficients – Equations with variable coefficients – Method of variation of parameters – Some standard methods –Laplace transforms.

Unit- III

Solutions in Power Series : Introduction – Second order linear equations with ordinary points – Legendre equation and Legendre Polynomials – Second order equations with regular singular points – Bessel functions.

Unit- IV

Oscillations of Second Order Equations: Introduction – Sturm's comparision theorem – Sturm's separation theorem-Elementary linear oscillations – Comparision theorem of Hille – Wintner – Oscillations of $x'' + a(t)x = 0$, Boundary value problems: Sturm – Liouville problem.

Text Book:

- **Ordinary Differential Equations** by S.G. Deo, V. Raghavendra , Rasmita Kar and V. Lakshmikantham , Third Edition, *McGraw-Hill Education(India)Private Limited, New Delhi.*

References:

1. **Differential Equations with Applications with Historical Notes** by George F.Simmons, *Second Edition.*
2. **Ordinary Differential Equations** by Earl A Coddington.

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M 104 **Semester-I**

Paper - IV: Elementary Number Theory

Unit- I

The Fundamental Theorem of Arithmetic: Divisibility- GCD- Prime numbers, Fundamental theorem of arithmetic- the series of reciprocal of the primes- The Euclidean algorithm.
(Page No. 13 - 23)

Unit- II

Arithmetical Functions and Dirichlet Multiplication: The functions $\phi(n)$, $\mu(n)$ and a relation connecting them- Product formula for $\phi(n)$ - Dirichlet product- Dirichlet inverse and Mobius inversion formula -The Mangoldt function $\wedge(n)$- Multiplicative functions and Dirichlet multiplication- The inverse of a completely multiplicative function- Liouville's function $\lambda(n)$- The divisor functions $\sigma_\alpha(n)$.
(Page No. 24-39 & 46-51)

Unit- III

Congruences: Properties of congruences- Residue classes and complete residue system- Linear congruences-Reduced residue systems and Euler-Fermat theorem- Polynomial congruence modulo p - Lagrange's theorem- Application of Lagrange's theorem- Chinese remainder theorem and its applications.
(Page No. 106-120 & 126-128)

Unit- IV

Quadratic Residues and The Quadratic Reciprocity Law: Quadratic residues- Legendre's symbol and its properties- Evaluation of $(-1|p)$ and $(2|p)$ - Gauss' lemma- The quadratic reciprocity law and its applications-The Jacobi symbol.
(Page No. 178-190 & 201-203)

Text Book:

- **Introduction to Analytic Number Theory** by Tom M. Apostol. *Narosa publishing house*

References:

1. **Number Theory** by Joseph H. Silverman.
2. **Theory of Numbers** by K.Ramchandra.
3. **Elementary Number Theory** by James K Strayer.
4. **Elementary Number Theory** by James Tattusall.

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M 201 **Semester-II**

Paper - I: Galois Theory

Unit- I

Algebraic extensions of fields: Irreducible polynomials and Eisenstein criterion - Adjunction of roots - Algebraic extensions - Algebraically closed fields.
(Page No. 281- 299).

Unit- II

Normal and separable extensions: Splitting fields - Normal extensions - Multiple roots - Finite fields - Separable extensions.
(Page No. 300 - 321).

Unit- III

Galois theory: Automorphism groups and fixed fields - Fundamental theorem of Galois theory - Fundamental theorem of Algebra.
(Page No. 322 - 339).

Unit- IV

Applications of Galois theory to classical problems: Roots of unity and cyclotomic polynomials - Cyclic extensions - Polynomials solvable by radicals – Symmetric functions-Ruler and Compass constructions.
(Page No. 340 - 364).

Text Book:

- **Basic Abstract Algebra** by P.B. Bhattacharya, S.K. Jain and S.R. Nagpaul. *Second Edition*

References:

1. **Topics in Algebra** by I.N. Herstein.
2. **Elements of Modern Algebra** by Gibert and Gilbert.
3. **Abstract Algebra** by Jeffrey Bergen.
4. **Basic Abstract Algebra** by Robert B Ash.

M 202 **Semester-II**

Paper - II: Lebesgue Measure and Integration

Unit- I

Algebra of sets - Borel sets - Outer measure - Measurable sets and Lebesgue measure - A non - measurable set - Measurable functions – Littlewood's three principles.

Unit- II

The Riemann integral - The Lebesgue integral of a bounded function over a set of finite measure - The integral of a non - negative function - The general Lebesgue integral.

Unit- III

Convergence in measure - Differentiation of monotone functions - Functions of bounded variation.

Unit- IV

Differentiation of an integral - Absolute continuity - The L_p - spaces - The Minkowski and Holder inequalities - Convergence and completeness.

Text Book:

- **Real Analysis** (3rd Edition)(Chapters 3, 4, 5) by H. L. Royden, *Prentice-Hall India.*

References:

1. **Lebesgue measure and Integration** by G.de Barra.
2. **Measure and Integral** by Richard L.Wheeden, Anotoni Zygmund.

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M 203 **Semester-II**

Paper III: Complex Analysis

Unit- I

Regions in the Complex Plane - Functions of a Complex Variable - Limits - Continuity - Derivatives - Cauchy – Riemann Equations - Sufficient Conditions for Differentiability - Analytic Functions - Harmonic Functions - Reflection Principle - The Exponential Function - The Logarithmic Function - Complex Exponents- Trigonometric functions- Hyperbolic functions .

Unit- II

Derivatives of Functions $w(t)$ - Definite Integrals of Functions $w(t)$ - Contours - Contour Integrals - Some Examples - Upper Bounds for Moduli of Contour Integrals – Anti derivatives - Cauchy – Goursat Theorem - Simply Connected Domains - Multiply Connected Domains - Cauchy Integral Formula - An Extension of the Cauchy Integral Formula - Liouville's Theorem and the Fundamental Theorem of Algebra - Maximum Modulus Principle.

Unit- III

Convergence of Sequences - Convergence of Series - Taylor Series - Laurent Series - Absolute and Uniform Convergence of Power Series - Isolated Singular Points - Residues - Cauchy's Residue Theorem - Residue at Infinity - The Three Types of Isolated Singular Points - Residues at Poles - Examples - Zeros of Analytic Functions - Zeros and Poles - Behavior of Functions Near Isolated Singular Points.

Unit- IV

Evaluation of Improper Integrals - Improper Integrals from Fourier Analysis - Jordan's Lemma - Definite Integrals Involving Sines and Cosines - Argument Principle - Rouche's Theorem - Linear Transformations - The Transformation $w = 1/z$ - Mappings by $1/z$ - Linear Fractional Transformations - An Implicit Form.

Text Book:

- **Complex Variables with Applications** by James Ward Brown and Ruel V Charcill. *McGraw- Hill International Edition.*

References:

1. **Complex Analysis** by Dennis G. Gill.
2. **Complex Analysis** by Steven G. Krantz.
3. **Complex Variables with Applications** by S. Ponnusamy, Herb Silverman.
4. **Complex Analysis** by Joseph Bak, Donald J. Newman.

M 204 **Semester-II**

Paper - IV: Integral Equations and Calculus of Variations

Unit- I

Volterra Integral Equations: Basic concepts - Relationship between Linear differential equations and Volterra Integral equations - Resolvent Kernel of Volterra Integral equation. Differentiation of some resolvent kernels - Solution of Integral equation by Resolvent Kernel - The method of successive approximations - Convolution type equations - Solution of Integro-differential equations with the aid of the Laplace Transformation – Volterra integral equation of the first kind-Euler integrals-Abel's problem-Abel's integral equation and its generalizations.

Unit- II

Fredholm Integral Equations : Fredholm integral equations of the second kind – Fundamentals – The Method of Fredholm Determinants - Iterated Kernels constructing the Resolvent Kernel with the aid of Iterated Kernels - Integral equations with Degenerated Kernels. Hammerstein type equation – Characteristic numbers and Eigen function and its properties.

Green's function :Construction of Green's function for ordinary differential equations-Special case of Green's function –Using Green's function in the solution of boundary value problem.

CALCULS OF VARIATIONS:

Unit- III

Introduction – The Method of Variations in Problems with fixed Boundaries: Definitions of Functionals –Variation and Its properties - Euler's'equation- Fundamental Lemma of Calculus of Variation – The problem of minimum surface of revolution - Minimum Energy Problem Brachistochrone Problem - Variational problems involving Several functions - Functional dependent on higher order derivatives - Euler Poisson equation.

Unit- IV

Functional dependent on the functions of several independent variables - Euler's equations in two dependent variables – Variational problems in parametric form-Applications of Calculus of Variation-Hamilton's principle - Lagrange's Equation,Hamilton's equations.

Text Book:

- **Problems and Exercises in Integral Equations** by M.KRASNOV, A.KISELEV, G.MAKARENKO, (1971).
- **Integral Equations** by S.Swarup, (2008).
- **Differential Equations and The Calculus of Variations** by L.ELSGOLTS, MIR Publishers, MOSCOW.